Weak solutions of elliptic problems in exterior domains

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Our aim is to obtain the existence of weak solution of the problem

$$\begin{cases} \triangle u(x) + f_1(x)f_2(u(x)) + g(x,u(x))x \cdot \triangledown u(x) = 0 \quad \text{for} \quad x \in \Omega_R \\ \lim_{\|x\| \to +\infty} u(x) = 0 \end{cases} \tag{1}$$

in $\Omega_R = \{x \in \mathbb{R}^n, \|x\| > R\}$ for $R > 1$ and $n \geq 5$. We assume that f_1 is a measurable function and there exists a sequence $(f_{1,m})_{m=1}^{\infty}$ which converges pointwise to f_1, while f_2 is a locally Hölder continuous function. Moreover, function g is bounded from above by $\tilde{g}(x)u^q$, where $\tilde{g} : \Omega_R \to [0, \infty)$ is such that $\tilde{g}(x)\|x\| \in L^2(\Omega_R)$ and $q > 3$. We also allow the case when $f_1(x)f_2(u)$ can be negative at the origin.

A starting point of this consideration is, for every $m \in \mathbb{N}$, the existence of a classical positive bounded solution u_m of (1) in Ω_R. Then we consider a certain auxilliary sequence of solutions $\tilde{u}_m$ (for every $m \in \mathbb{N}$) and show that $\tilde{u}_m$ has a subsequence converging weakly to some $u_0 \in W_0^{1,2}$, which is a weak solution of (1).

References

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